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Deriving Vertices, Shape Functions for Elliptic Duct Geometry and Verified Two Verification Conditions

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Abstract— In this paper we derived vertices for elliptic duct geometry and derived shape functions for elliptic duct geometry and verified the conditions that sum of all the shape functions is equal to one and each shape function has a value of one at its own node and zero at the other nodes.

Index Terms—Shape functions, elliptic duct geometry, Vertices.

1 INTRODUCTION

One of the powerful methods to analyse and understand any real phenomenon is to formulate the best suited mathematical model based on certain hypothesis like continuum hypothesis etc., which can can be solved making use of either exact methods or possible approximate methods. The solvability of the boundary value problem depends on the nature of the equation as well as the shape of the boundaries involved [2]. Among the different numerical techniques, the variational methods, weighted residual methods and finite difference methods, finite element methods are quite popular. Some standard finite element books are [1,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18].

2 GEOMETRICAL DESCRIPTION

The Cartesian system is chosen with origin on the central axis of the duct and the axis O(x', y') are parallel to its major axes of its crosssection shown in schematic diagram of elliptic duct.We divide the domain into sub-domains of our choice. The sub-domains may be lines or triangles or rectangles etc, depending on the boundary of the configuration.This division of the domain is usually known as mesh generation and each sub-domain is called an element of the mesh. Some of the elements may be of different shape compare to the rest in the mesh, how ever care should be taken that all the elements have known geometrical shapes. Also the number of elements involved differ from problem to problem and depends on thepercentage of accuracy of the solution. We analyze the convective heat and mass transfer flow of a viscous, incompressible fluid through a porous medium in an elliptic duct. The flow being symmetric with respect to the vertical diameter x′=0, theanalysis is carried out in the right semicircular duct. For computation purpose we descritized the domain into 14 triangular elements and each triangular element is 6 noded. Also total number of nodes in right semicircular duct is 39 nodes.

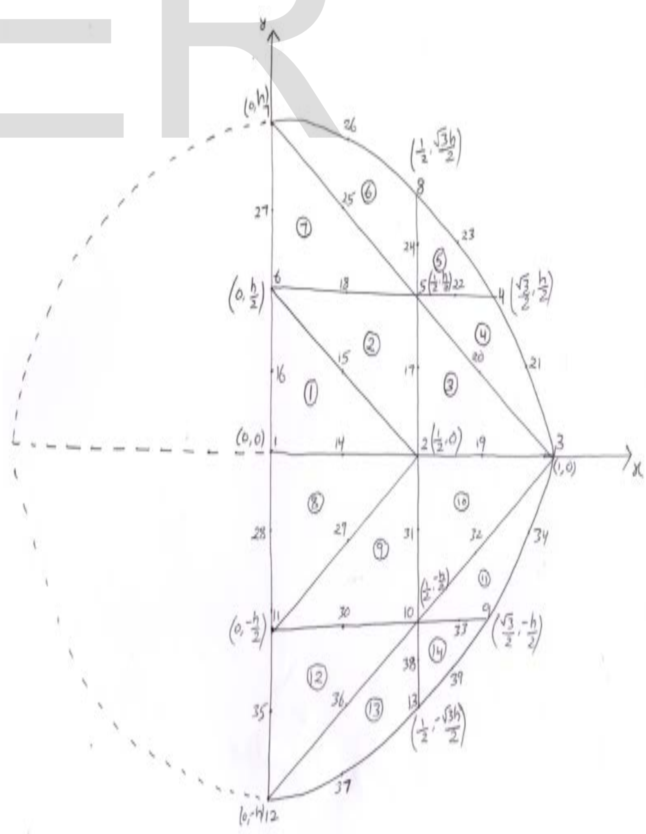

Fig.1 Schematic diagram of Elliptic Duct

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(I) Deriving Vertices for Elliptic Duct Geometry

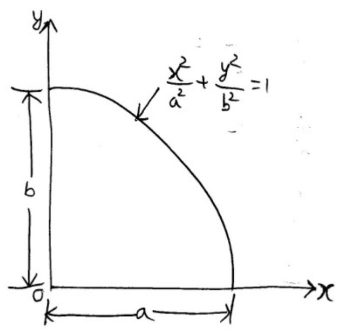

Fig. 2 Quarter Ellipse

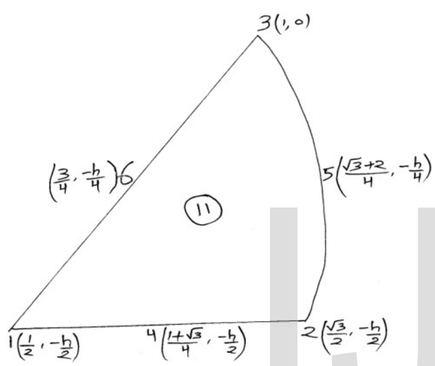

Fig. 3 Element 11 (6 noded)

In Schematic diagram of elliptic duct, length of right semi Circular duct along x-axis is 1 i.e., a=1

Length of right semi circular duct along y-axis is h i.e., b=h.

Ellipse equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ....(1)$$

Along x - axis, y = 0

$$(1) \Rightarrow \frac{x^2}{a^2}+\frac{(0)^2}{b^2}=1 \Rightarrow \frac{x^2}{a^2}+\frac{0}{b^2}=1 \Rightarrow \frac{x^2}{a^2}+0=1$$

$$\Rightarrow \frac{x^2}{a^2}=1 \Rightarrow x^2=a^2 \Rightarrow x=\pm\sqrt{a^2}$$

$$\Rightarrow x=\pm a \qquad (\because \text{a}=1) \Rightarrow x=\pm 1$$

Vertices are (1,0) and (-1,0)

Along y - axis, x = 0

$$(1) \Rightarrow \frac{(0)^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow 0+\frac{y^2}{b^2}=1 \Rightarrow \frac{y^2}{b^2}=1$$

$$\Rightarrow y^2=b^2 \Rightarrow y=\pm\sqrt{b^2} \Rightarrow y=\pm b \quad (\because \text{b}=\text{h})$$

$$\Rightarrow y=\pm h$$

Vertices are (0,h) and (0,-h)

At center of the ellipse vertex is (0,0) and end vertex along x-axis is (1,0)

Midpoint of (0,0) and (1,0)

Midpo int formula for (x_1, y_1) and (x_2, y_2)

is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpo int of $\overset{(x_1,y_1)}{(0,0)}$ and $\overset{(x_2,y_2)}{(1,0)}$

$$=(\frac{0+1}{2},\frac{0+0}{2})=(\frac{1}{2},0)$$

Mid Point of (0,0) and (1,0) is $\left(\frac{1}{2},0\right)$

At center of the ellipse vertex is (0,0) and the vertex along y-axis is (0,h) and (0,-h)

Midpo int of $\overset{(x_1,y_1)}{(0,0)}$ and $\overset{(x_2,y_2)}{(0,h)}$

$$=(\frac{0+0}{2},\frac{0+h}{2})=(0,\frac{h}{2})$$

Mid Point of (0,0) and (0,h) is $\left(0,\frac{h}{2}\right)$

Midpo int of $\overset{(x_1,y_1)}{(0,0)}$ and $\overset{(x_2,y_2)}{(0,\text{-h})}$

$$=(\frac{0+0}{2},\frac{0+(-h)}{2})=(0,\frac{-h}{2})$$

$\therefore$ *Mid* vertex of (0,0) and (0,-h) is $(0,\frac{\text{-h}}{2})$

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$$\begin{array}{lcccc} & (x_1,y_1) & & (x_2,y_2) \\ \textit{Midpoint of} & (0,h) & \text{and} & (1,0) \end{array}$$

$$= \left(\frac{0+1}{2}, \frac{h+0}{2}\right) = \left(\frac{1}{2}, \frac{h}{2}\right)$$

$$\begin{array}{lcccc} & (x_1,y_1) & & (x_2,y_2) \\ \textit{Midpoint of} & (0,-h) & \text{and} & (1,0) \end{array}$$

$$= \left(\frac{0+1}{2}, \frac{-h+0}{2}\right) = \left(\frac{1}{2}, \frac{-h}{2}\right)$$

In eq(1) put $x = \frac{1}{2}$

$$(1) \Rightarrow \frac{(\frac{1}{2})^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (\because a = 1, b = h)$$

$$\Rightarrow \frac{\frac{1}{4}}{(1)^2} + \frac{y^2}{h^2} = 1 \Rightarrow \frac{1}{4} + \frac{y^2}{h^2} = 1 \Rightarrow \frac{y^2}{h^2} = 1 - \frac{1}{4}$$

$$\Rightarrow \frac{y^2}{h^2} = \frac{3}{4} \Rightarrow y^2 = \frac{3}{4}h^2 \Rightarrow y = \pm\sqrt{\frac{3}{4}h^2} \Rightarrow y = \pm\frac{\sqrt{3}}{2}h$$

$\therefore End$ vertex along y-axis at $x = \frac{1}{2}$ is $(\frac{1}{2}, \frac{\sqrt{3}}{2}h)$

and $(\frac{1}{2}, \frac{-\sqrt{3}}{2}h)$

In eq(1) put $y = \frac{h}{2}$

$$(1) \Rightarrow \frac{x^2}{a^2} + \frac{(\frac{h}{2})^2}{b^2} = 1 \; (\because a = 1,\; b = h) \Rightarrow \frac{x^2}{(1)^2} + \frac{\frac{h^2}{4}}{(h)^2} = 1$$

$$\Rightarrow \frac{x^2}{1} + \frac{\frac{h^2}{4}}{h^2} = 1 \Rightarrow \frac{x^2}{1} + \frac{h^2}{4} X \frac{1}{h^2} = 1$$

$$\Rightarrow x^2 + \frac{1}{4} = 1 \Rightarrow x^2 = 1 - \frac{1}{4} \Rightarrow x^2 = \frac{3}{4} \Rightarrow \sqrt{x^2} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}$$

$\therefore End$ vertex along x-axis at $y = \frac{h}{2}$ is $(\frac{\sqrt{3}}{2}, \frac{h}{2})$

In eq(1) put $y = \frac{-h}{2}$

$$(1) \Rightarrow \frac{x^2}{a^2} + \frac{(\frac{-h}{2})^2}{b^2} = 1 \qquad (\because a = 1,\; b = h)$$

$$\Rightarrow \frac{x^2}{(1)^2} + \frac{\frac{h^2}{4}}{(h)^2} = 1 \Rightarrow \frac{x^2}{1} + \frac{\frac{h^2}{4}}{h^2} = 1 \Rightarrow$$

$$\frac{x^2}{1} + \frac{h^2}{4} X \frac{1}{h^2} = 1 \Rightarrow x^2 + \frac{1}{4} = 1 \Rightarrow x^2 = 1 - \frac{1}{4}$$

$$\Rightarrow x^2 = \frac{3}{4} \Rightarrow \sqrt{x^2} = \sqrt{\frac{3}{4}} \Rightarrow x = \frac{\sqrt{3}}{2}$$

$\therefore End$ vertex along x-axis at $y = \frac{-h}{2}$ is $(\frac{\sqrt{3}}{2}, \frac{-h}{2})$

(II) Deriving Shape functions for Elliptic Duct Geometry

For element 11 vertices at corner nodes (Node 1, Node 2, Node3)

(*Element 11*)

$Node\ 1 = (x_1, y_1) \quad Node\ 2 = (x_2, y_2) \quad Node\ 3 = (x_3, y_3)$

$$x_1 := \frac{1}{2} \quad y_1 := -\frac{h}{2} \quad x_2 := \frac{\sqrt{3}}{2} \quad y_2 := -\frac{h}{2} \quad x_3 := 1 \quad y_3 := 0$$

For element 11 vertices at mid nodes (node 4, node 5, node 6)

(*Element 11*)

$Node\ 4 = (x_4, y_4)$

$$x_4 := \frac{1+\sqrt{3}}{4} \qquad y_4 := -\frac{h}{2}$$

$$(\textit{Mid}\ \text{Point of}\ (\frac{1}{2}, \frac{-h}{2})\ \&\ (\frac{\sqrt{3}}{2}, \frac{-h}{2}) = (\frac{\frac{1}{2}+\frac{\sqrt{3}}{2}}{2}, \frac{\frac{-h}{2}+(\frac{-h}{2})}{2})$$

$$= (\frac{1+\sqrt{3}}{4}, \frac{-h}{2}))$$

$Node\ 5 = (x_5, y_5)$

$$x_5 := \frac{\sqrt{3}+2}{4} \qquad y_5 := -\frac{h}{4}$$

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(*Mid* Point of ($\frac{\sqrt{3}}{2}, \frac{-h}{2}$) & (1, 0)

$$= \left(\frac{\frac{\sqrt{3}}{2}+1}{2}, \frac{\frac{-h}{2}+0}{2}\right) = \left(\frac{\sqrt{3}+2}{4}, \frac{-h}{4}\right))$$

$$Node\ 6=(x_6, y_6)$$

$$x_6 := \frac{3}{4} \quad y_6 := -\frac{h}{4}$$

(*Mid* Point of ($\frac{1}{2}, \frac{-h}{2}$) & (1, 0)

$$= \left(\frac{\frac{1}{2}+1}{2}, \frac{\frac{-h}{2}+0}{2}\right) = \left(\frac{3}{4}, \frac{-h}{4}\right))$$

Shape function formulae for six noded triangular element

$$n_1 := p*(2*p-1) \qquad n_2 := q*(2*q-1)$$

$$n_3 := r*(2*r-1) \qquad n_4 = 4*p*q$$

$$n_5 = 4*q*r \qquad n_6 = 4*p*r$$

Where

$$p := ((x_2 * y_3 - x_3 * y_2) + x*(y_2 - y_3) + y*(x_3 - x_2))/Det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

$$q := ((x_3 * y_1 - x_1 * y_3) + x*(y_3 - y_1) + y*(x_1 - x_3))/Det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

$$r := ((x_1 * y_2 - x_2 * y_1) + x*(y_1 - y_2) + y*(x_2 - x_1))/Det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

$$Do[\Pr int[S_i = n_i], \{i, 1, 6\}]$$

Output

Shape functions for element 11

$$s_1 == \frac{\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)\left(-1+\frac{2\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}} \tag{2}$$

$$s_2 == \frac{\left(-1+\frac{2\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}\right)\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}} \tag{3}$$

$$s_3 == \frac{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)\left(-1+\frac{2\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}} \tag{4}$$

$$s_4 == \frac{4\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)}{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}\right)^2} \tag{5}$$

$$s_5 == \frac{4\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)}{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}\right)^2} \tag{6}$$

$$s_6 == \frac{4\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)}{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}\right)^2} \tag{7}$$

(III) Verification : Sum of all the Shape Functions is equal to One

$$s_5 == \frac{4\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)}{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}\right)^2}$$

$$s_1 := \frac{\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)\left(-1+\frac{2\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}$$

$$s_6 := \frac{4\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)}{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}\right)^2}$$

$$s_2 := \frac{\left(-1+\frac{2\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}\right)\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}$$

$$s_3 := \frac{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)\left(-1+\frac{2\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)y\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}\right)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}$$

$$s_4 := \frac{4\left(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2}\right)\left(\frac{h}{2}-\frac{hx}{2}+\left(1-\frac{\sqrt{3}}{2}\right)y\right)}{\left(-\frac{h}{4}+\frac{\sqrt{3}h}{4}\right)^2}$$

$$Simplify[s_1+s_2+s_3+s_4+s_5+s_6]$$

Output
1

(IV). Verification: Each shape function has a value of one at its own node and zero at the other nodes

(i) In Element 11 at the vertex $(\frac{1}{2},-\frac{h}{2})$ **(Node 1)**

Substituting $x:=\frac{1}{2}$, $y:=-\frac{h}{2}$ in equations (2), (3), (4), (5), (6) and (7)

$$s_1 := \frac{(\frac{h}{2}-\frac{hx}{2}+(1-\frac{\sqrt{3}}{2})y)(-1+\frac{2(\frac{h}{2}-\frac{hx}{2}+(1-\frac{\sqrt{3}}{2})y)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}})}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}$$

$$s_2 := \frac{(-1+\frac{2(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2})}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}})(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2})}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}$$

$$s_3 := \frac{(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+(-\frac{1}{2}+\frac{\sqrt{3}}{2})y)(-1+\frac{2(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+(-\frac{1}{2}+\frac{\sqrt{3}}{2})y)}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}})}{-\frac{h}{4}+\frac{\sqrt{3}h}{4}}$$

$$s_4 := \frac{4(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2})(\frac{h}{2}-\frac{hx}{2}+(1-\frac{\sqrt{3}}{2})y)}{(-\frac{h}{4}+\frac{\sqrt{3}h}{4})^2}$$

$$s_5 == \frac{4(-\frac{h}{2}+\frac{hx}{2}-\frac{y}{2})(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+(-\frac{1}{2}+\frac{\sqrt{3}}{2})y)}{(-\frac{h}{4}+\frac{\sqrt{3}h}{4})^2}$$

$$s_6 := \frac{4(\frac{h}{2}-\frac{hx}{2}+(1-\frac{\sqrt{3}}{2})y)(-\frac{h}{4}+\frac{\sqrt{3}h}{4}+(-\frac{1}{2}+\frac{\sqrt{3}}{2})y)}{(-\frac{h}{4}+\frac{\sqrt{3}h}{4})^2}$$

$Simplify[s_1]$ $Simplify[s_2]$ $Simplify[s_3]$
$Simplify[s_4]$ $Simplify[s_5]$ $Simplify[s_6]$

Output for element at the vertex $(\frac{1}{2},-\frac{h}{2})$
1
0
0
0
0
0

(ii) In Element 11 at the vertex $(\frac{\sqrt{3}}{2},-\frac{h}{2})$ **(Node 2)**

(*Node 2*)

Substituting $x:=\frac{\sqrt{3}}{2}$, $y:=-\frac{h}{2}$ in equations (2), (3), (4), (5), (6) and (7)

$Simplify[s_1]$ $Simplify[s_2]$ $Simplify[s_3]$
$Simplify[s_4]$ $Simplify[s_5]$ $Simplify[s_6]$

Output for element at the vertex $(\frac{\sqrt{3}}{2},-\frac{h}{2})$
0
1
0
0
0
0

(iii) In Element 6 at the vertex $(1,0)$ **(Node 3)**

Substituting x:=1, y:=0 in equations (2), (3), (4), (5), (6) and (7)

$Simplify[s_1]$ $Simplify[s_2]$ $Simplify[s_3]$
$Simplify[s_4]$ $Simplify[s_5]$ $Simplify[s_6]$

Output for element at the vertex $(1,0)$
0
0
1
0
0
0

(iv) In Element 11 at the vertex $(\frac{1+\sqrt{3}}{4},-\frac{h}{2})$ **(Node 4)**

Substituting $x:=\frac{1+\sqrt{3}}{4}$, $y:=-\frac{h}{2}$ in equations (2), (3), (4), (5), (6) and (7)

$Simplify[s_1]$ $Simplify[s_2]$ $Simplify[s_3]$
$Simplify[s_4]$ $Simplify[s_5]$ $Simplify[s_6]$

Output for element at the vertex $(\frac{1+\sqrt{3}}{4},-\frac{h}{2})$

0
0
0
1
0
0

(v) In Element 11 at the vertex (Node 5) $(\frac{\sqrt{3}+2}{4},-\frac{h}{4})$

Substituting x:= $\frac{\sqrt{3}+2}{4}$, y:= $-\frac{h}{4}$ in equations (2), (3), (4), (5), (6) and (7)

$Simplify[s_1]$ $Simplify[s_2]$ $Simplify[s_3]$
$Simplify[s_4]$ $Simplify[s_5]$ $Simplify[s_6]$

Output for element at the vertex $(\frac{\sqrt{3}+2}{4},-\frac{h}{4})$

0
0
0
0
1
0

(vi) In Element 11 at the vertex (Node 6) $(\frac{3}{4},-\frac{h}{4})$

Substituting x:= $\frac{1}{2}$, y:= $\frac{h+\sqrt{3}h}{4}$ in equations (2), (3), (4), (5), (6) and (7)

$Simplify[s_1]$ $Simplify[s_2]$ $Simplify[s_3]$
$Simplify[s_4]$ $Simplify[s_5]$ $Simplify[s_6]$

Output for element at the vertex $(\frac{3}{4},-\frac{h}{4})$

0
0

0
0
0
1

3. CONCLUSIONS

1. Derived Vertices for Elliptic Duct Geometry
2. Derived Shape Functions for Elliptic Duct Geometry
3. Verified the condition that Sum of all the Shape functions is equal to one
4. Verified the condition that each shape function has a value of one at its own node and zero at the other nodes.

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